Stronger Neutrino Interactions at Extremely High Energies and the Muon Anomalous Magnetic Moment

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Abstract

A specific model of parity-conserving lepton substructure is considered. We show that a positive-definite contribution to the muon $(g-2)/2$ at the possible level of about 4×10^{-9}, can be related to a significant increase in the interaction cross section for cosmic-ray neutrinos with energies above about 10^{19} eV. The additional cross section at $\sim 10^{20}$ eV is calculated to be $\sim 10^{-29}$ cm^2, which is about 100 times the standard weak-interaction cross section. The model involves an extremely massive, neutral lepton, with $m_L \cong 2\times10^6$ GeV fixed by the new contribution to $(g-2)/2$.

One may consider a recently measured small deviation[1] from the theoretically-calculated value of the muon anomalous magnetic moment as an indication of possible new physics.[2] In doing so, it would be valuable if two conditions were fulfilled.

(1) The sign of the experimental deviation is positive. The new physics should produce this sign unambiguously. That is, it should contribute to the anomalous magnetic moment a positive-definite quantity.

(2) It would be useful if the new physics implied other definite physical processes which are directly calculable. Further, these processes should be accessible to exploration in current experiments. They may well be related to puzzling aspects of the present experimental situation in different areas.

In this paper, we give and discuss the results of straight-forward calculations based upon hypothetical additional^{F1} dynamics which relates the possible deviation in $(g-2)/2$ for the muon to the possible occurrence of stronger-than-weak interactions for muon (and tau) neutrinos at very high energies, $E_\nu > 10^{19}$ eV. The latter possibility concerns the nature and origin of the particles which initiate the highest-energy cosmic-ray air showers, with energies up to about 10^{20} eV. This issue will be under further intense study in on-going[3, 4] and up-coming cosmic-ray experiments [5, 6].

In the early classic review of QED[7], a variety of contributions to the muon $(g-2)/2$ were calculated from hypothetical couplings to "exotic particles" (section

^{F1} That is, additional to the dynamics contained in QED and in the standard electroweak-QCD model.

7.3). A positive-definite result[7] is obtained for the process shown in the Feynman diagram in Fig. 1,

$$\Delta a_\mu = \frac{g^2}{4\pi}\frac{1}{4\pi}\left(\frac{m_\mu}{m_L}\right)\left\{1 - \frac{2}{3}\left(\frac{m_\mu}{m_L}\right)\right\}, \quad m_L \gg m_\mu \sim m_\pi \tag{1}$$

In Fig. 1, L denotes a very massive, spin-1/2 (muonic) neutral lepton which is assumed to be coupled to a muon and a hypothetical pointlike component of a pion (a Goldstone component?) with a strong parity-conserving, effective Hermitian interaction of the form

$$ig\left\{(\overline{L}\gamma_5\mu)\pi^+ + (\overline{\mu}\gamma_5 L)\pi^-\right\}, \quad \gamma_5^\dagger = \gamma_5,\ g \text{ real} \tag{2}$$

where the particle symbols stand for the fields. With $g^2/4\pi \sim 1$ and[1] $\Delta a_\mu \sim 4 \times 10^{-9}$, Eq. (1) immediately gives a very large mass,

$$m_L \cong 2 \times 10^6 \text{ GeV} \tag{3}$$

The general physical idea[8] embodied phenomenologically in Eq. (2) is that of a parity-conserving substructure in leptons, a structure which is very compact in space because of an effectively very massive "core" region. It is not necessary to augment this idea with specific theoretical details in order to go immediately to a further unusual new consequence for certain experiments at the highest energies. Consider the analogue of Eq. (2) involving a neutrino ν_μ,

$$ig\left\{(\overline{L}\gamma_5\nu_\mu)\pi^0 + (\overline{\nu}_\mu\gamma_5 L)\pi^0\right\} \tag{4}$$

For a center-of-mass energy $\sqrt{s} \cong m_L \cong 2\times 10^6$ GeV in a neutrino-nucleon collision in the atmosphere, the neutrino energy must be $E_\nu \sim (s/2m_{\text{N}}) \cong 2 \times 10^{21}$ eV. This energy is a factor of about 10 above that of the present small number[3, 4] of highest-energy cosmic-ray air showers, which are estimated to be at, or just above, 10^{20} eV. However, the process[F2] shown in the Feynman diagram in Fig. 2 gives rise to an additional neutrino interaction cross section in the atmosphere. Here the L contributes virtually; the final-state products are a lepton and a pion. In addition, there is the multi-hadron production from the pion total absorption cross section which controls the strength of the squared lower vertex from Fig. 1. What is particularly interesting is the result of our calculation of the additional neutrino cross section, shown in the graph in Fig. 3. For $m_L \cong 2\times 10^6$ GeV (fixed by Δa_μ with a strong $g^2/4\pi \sim 1$), $\Delta\sigma_\nu(\sqrt{s})$ is comparable to a standard weak-interaction cross section of the order of 10^{-31} cm^2 at $E_\nu \sim 10^{19}$ eV, but steadily rises to a considerably larger value of $\sim 2.7\times 10^{-29}$ cm^2 at $E_\nu \sim 2\times 10^{20}$ eV. This rise is of course, just what is expected for the approach to production of real L, as E_ν increases (but possibly never reaches $\sim 2\times 10^{21}$ eV, for actual cosmic-ray neutrinos[9]). The expression for $\Delta\sigma_\nu(\sqrt{s})$ giving the curve in Fig. 3 is (in millibarns),

$$\begin{aligned}\Delta\sigma_\nu(\sqrt{s}) &= \left(\frac{g^2}{4\pi}\right)^3 \frac{4\sigma_{\pi\text{N}}^{\text{tot}}}{\pi}\int_0^{\frac{\sqrt{s}}{2}} dL(L^2)\int_0^{\frac{(s-2\sqrt{s}L)^{1/2}}{2}} dQ(Q^2)\gamma \\ &\times \int_{-1}^{1} dx\frac{\left(\frac{s}{4}+L^2-\sqrt{s}Lx\right)^{1/2}(\gamma 2Q - Lx)}{(4Q^2-m_L^2)^2(4Q^2-\sqrt{s}(\gamma 2Q-Lx)-m_\pi^2)^2} \\ &\times \frac{(1-v)}{v}\ln\left(\frac{1+v}{1-v}\right)\end{aligned} \tag{5}$$

[F2] An analogous process involves an extremely high-energy incident muon.

with $\gamma = 1/(1-v^2)^{1/2}$, $v = L/(4Q^2+L^2)^{1/2}$, $m_L = 2\times 10^6$ GeV.
The cross section $\sigma_{\pi \mathrm{N}}^{\mathrm{tot}}$ is approximated by a constant of the order of 200 mb.[10]. The squared invariant mass in the final lepton-pion system is $4Q^2$, L is the magnitude of the three-momentum of the virtual, massive lepton; this three-momentum makes an angle θ ($x = \cos\theta$) with the incident neutrino direction in the c. m. system of the neutrino-nucleon collision. Masses of nucleon and final lepton and pion are neglected in the kinematics; the pion mass is included in the pion propagator to formally avoid singular behavior. Clearly, with an interaction cross section of the order of 100 times greater than that from the electroweak interaction, cosmic-ray neutrinos with $E_\nu \sim 10^{20}$ eV have a significantly greater probability to produce detectable air showers, if there is sufficient flux. However, $\Delta\sigma_\nu$ is probably still not large enough to account for the (presently few, ~ 14) highest-energy events[4] coming from small zenith angles[3] ($< 45°$). This might be possible if there is a very high energy neutrino flux as large ^{F3} [9] as about $10^{-17}(\mathrm{cm}^2 - \mathrm{s} - \mathrm{sr})^{-1}$, and an interaction probability in air as large as about 10^{-3}. But there must be air showers at 10^{20} eV coming from large zenith angles. Recently, two such showers have been reported and analyzed.[11] The study of the frequency and of the characteristics of air showers at large zenith angles is expected to increase. The Auger air-shower array[5] will augment the present sensitivity for "horizontal" neutrino-induced air showers at $E_\nu \sim 10^{20}$ eV, down to a possible flux of about 10^{-17} $(\mathrm{cm}^2{-}\mathrm{s}{-}\mathrm{sr})^{-1}$. The Antarctic ice detector specifically for neutrinos (and muons), when in expanded operation, is said[6] also to be able to detect a flux of neutrinos as low as $10^{-17}(\mathrm{cm}^2 - \mathrm{s} - \mathrm{sr})^{-1}$ at $E_\nu \sim 10^{20}$ eV.

There is a further unusual physical process that can be envisioned, and estimated in the context of the above ideas^{F4} : this is $\tau \to \mu + \gamma$. Consider two L particles, L_1 and L_2 with masses m_{L_1} and m_{L_2}, which mix with an angle θ_L in the flavor states

$$\begin{aligned} L_\mu &= (L_1 \cos\theta_L + L_2 \sin\theta_L) \\ L_\tau &= (L_2 \cos\theta_L - L_1 \sin\theta_L) \end{aligned} \tag{6}$$

The effective interaction analogous to Eq. (4) is

$$ig\left\{(\overline{L}_\mu \gamma_5 \nu_\mu)\pi^0 + (\overline{L}_\tau \gamma_5 \nu_\tau)\pi^0 + h.c.\right\} \tag{7}$$

The branching ratio for $\tau \to \mu + \gamma$ is then,

$$B.R.(\tau \to \mu + \gamma) \cong \frac{\frac{\alpha}{4}|\tilde{\Delta a_\mu}|^2 m_\tau}{(\tau_\tau)^{-1}} \sim 1.5 \times 10^{-9} \tag{8}$$

where the τ lifetime is $\tau_\tau \cong 2.9 \times 10^{-13}$ s, the mass is $m_\tau \cong 1.78$ GeV, and $\alpha = 1/137$. The quantity $|\tilde{\Delta a_\mu}|$ is

$$|\tilde{\Delta a_\mu}| \sim (m_L \Delta a_\mu) \times \left|(\cos\theta_L \sin\theta_L)\frac{(m_{L_2} - m_{L_1})}{m_{L_1} m_{L_2}}\right| \sim \frac{\Delta a_\mu}{4} \tag{9}$$

where we have <u>assumed</u>, simply for illustration, $|(\cos\theta_L \sin\theta_L) m_L (m_{L_2} - m_{L_1})/(m_{L_1} m_{L_2})|$ to be of the order of $1/(2)^2$ i. e. for non-zero $|\theta_L| \sim \pi/4$, and with $m_{L_2} \sim \frac{m_{L_1}}{1.5} = \frac{m_L}{1.5}$.^{F4} The present experimental upper limit for this branching ratio is 1.1×10^{-6}.

^{F3} A flux of τ neutrinos of up to about $10^{-16}(\mathrm{cm}^2 - \mathrm{s} - \mathrm{sr})^{-1}$ has been estimated in [9, 12] as arising from the <u>two-body</u> decay $\phi \to \nu_\tau \overline{\nu}_\tau$, of a metastable, scalar inflaton (lifetime of $\sim 10^{26}$s because the decay amplitude is proportional to the neutrino mass). The ϕ mass has been calculated, from the minimum of the potential, to be of the order of 10^{11} GeV. [12, 13]. The ϕ can constitute a significant part of dark matter throughout the present universe.[12]

^{F4} Note that a contribution to the electron $(g-2)/2$ of about $(m_e/m_\mu)\frac{1}{2}(\Delta a_\mu) \sim (1/400)(\Delta a_\mu)$ is marginally allowed by the present uncertainty (i. e. via an electron L_3 with $m_{L_3} \sim 2m_L$). But absence of $\mu \to e\gamma$, requires essentially no mixing of L_3.

In summary, a strong, parity-conserving interaction of a colorless quark composite like a pion with a muon and a very massive neutral lepton, gives a positive-definite contribution to the muon anomalous magnetic moment of about 4×10^9, when the massive lepton mass m_L is about 2×10^6 GeV. An analogous interaction for neutrinos[F5] leads directly to an additional interaction cross section for very high energy cosmic-ray neutrinos in the atmosphere. As a result of the above explicit value of m_L extracted from the present discrepancy in a_μ[1], and the explicit dynamics in Fig. 2, computation gives a $\Delta\sigma_\nu(\sqrt{s})$ which surprisingly, becomes comparable to the standard weak-interaction cross section of about 10^{-31} cm^2 at $E_\nu = 10^{19}$ eV. The calculated result (Fig. 3) that $\Delta\sigma_\nu(\sqrt{s})$ increases strongly[F6], to about 100 times σ_{weak} at $E_\nu \sim 10^{20}$ eV, opens the possibility for an observable neutrino-induced component in the highest-energy cosmic-ray air showers, if the neutrino flux is as high[9] as $10^{-17}(\text{cm}^2 - \text{s} - \text{sr})^{-1}$. Thus, experiments[1, 4, 5, 6] on the muon $(g-2)/2$ and on the interaction of extremely high-energy neutrinos, may be considered as related probes for a definite kind of lepton substructure at extraordinarily small distances.[F7]

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[F5] Other possible effects, such as an induced, neutrino (and L) magnetic moment ($O(10^{-11}\mu_B)$), although interesting, appear to be well below present experimental upper limits.

[F6] The sensitivity to m_L in Eq. (5) is such that $m_L \to m_L/1.5$, increases the $\Delta\sigma_\nu$ in Fig. 3 by about 5. With exclusive reference to τ neutrinos, an $m_{L_2} \sim m_L/3$ yields a $\Delta\sigma_{\nu_\tau} \sim 10^{-27}$ cm^2 at $\sim 10^{20}$ eV; the atmospheric interaction of ν_τ would become hadron-like.

[F7] In this model with γ_5 coupling, a radiative correction can give a downward shift from a "bare" lepton mass.[2]

Figures

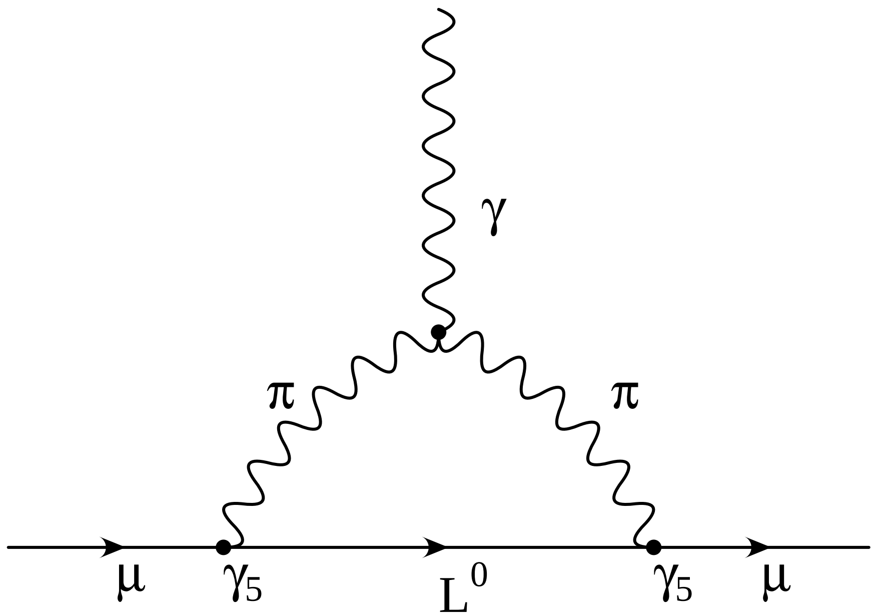

Figure 1: Feynman diagram for a contribution to the muon anomalous magnetic moment due to a strong, parity-conserving interaction with a very massive, neutral lepton L^0.

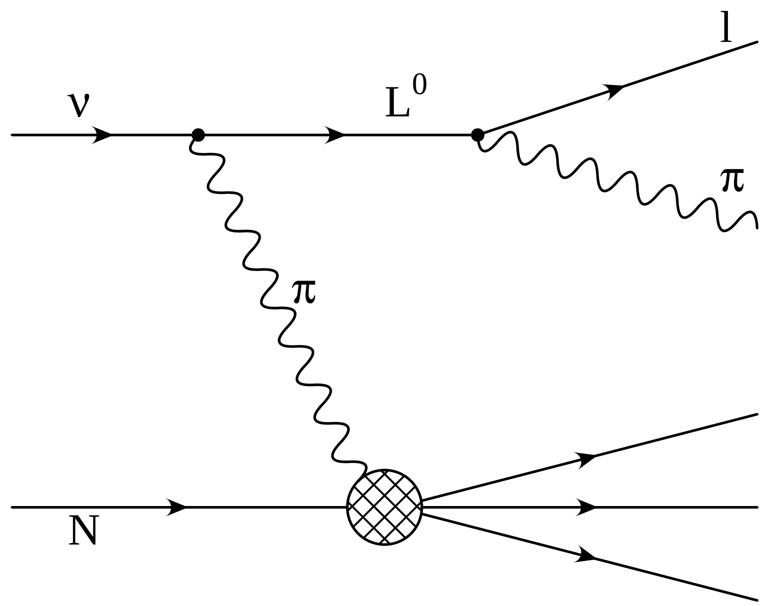

Figure 2: Feynman diagram for a contribution to the interaction cross section of an extremely high energy, cosmic-ray neutrino with an atmospheric nucleon, mediated by a virtual L^0.

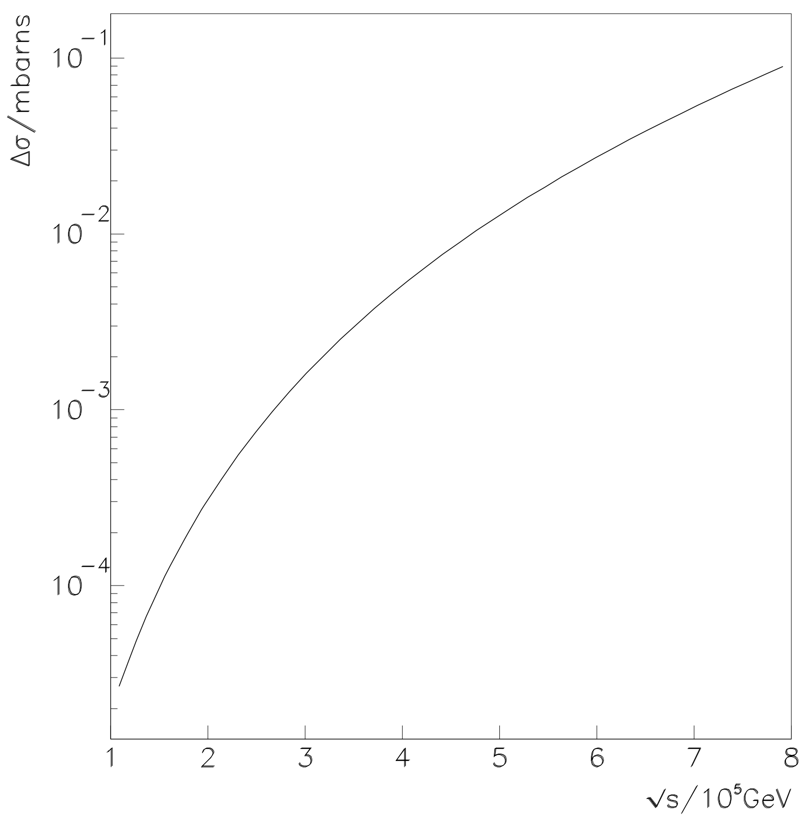

Figure 3: The additional neutrino-nucleon interaction cross section $\Delta\sigma(\sqrt{s})$ calculated from Eq. (5), plotted versus the c. m. energy $\sqrt{s}$. The curve is for cosmic-ray neutrinos with E_ν from $\sim 10^{19}$ eV to $\sim 3 \times 10^{20}$ eV.